

UNDERMATCHING ON CONCURRENT VARIABLE-INTERVAL SCHEDULES AND THE POWER LAW

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The phenomenon of undermatching on concurrent variable-interval schedules is shown to be derivable by transforming the individual interreinforcement intervals of each variable-interval schedule and averaging the transformed values to produce an "estimate" of the rate of reinforcement the schedules deliver. If the transformation is based on a power function with a fractional exponent, such as is found in many studies of temporal control in animals, matching response ratios to the ratios of these estimated rates of reinforcement yields undermatching. If the concurrent variable-interval schedules are arranged such that the individual intervals in each schedule have a constant proportionality (a procedure found in many commonly used variable-interval schedules) the slope of the line relating logarithms of response ratios and of programmed reinforcement ratios is identical to the exponent of the power transformation applied to the individual time intervals in the variable-interval schedules. In other cases this simple relation does not hold but the degree of undermatching is greater the lower the value of the exponent of the power function. This account of undermatching predicts values similar to those typically observed.

Key words: concurrent variable-interval schedules, undermatching, power law, time estimation

Herrnstein (1961) reported that pigeons, when confronted with two concurrently presented variable-interval schedules (*conc VI VI*), distributed their responses to the two response alternatives according to

$$\frac{R_a}{R_a + R_b} = \frac{r_a}{r_a + r_b} \quad (1)$$

or, equivalently,

$$\frac{R_a}{R_b} = \frac{r_a}{r_b} \quad (2)$$

where R_a and R_b are the rates of responding to schedules a and b , and r_a and r_b the rates of reinforcer delivery (e.g., in reinforcers per hour).

More recent research has suggested, however, that the above formulation (usually called the Matching Law) may not generally hold. Baum (1974) proposed that two types of deviation from matching might occur. The first of these (bias) is a consistent preference for one response alternative over another and is usually considered (Baum, 1974; de Villiers, 1977) to reflect the influence of unmeasured

sources of reinforcement in the experimental situation. The second (undermatching) is a larger preference for the schedule delivering the lower rate of reinforcement than the Matching Law predicts. Reanalysis of earlier data (e.g., by de Villiers, 1977, and Myers & Myers, 1977) found bias to be generally present and found frequent undermatching, particularly when preference was measured in terms of numbers of responses emitted rather than time allocation.

This suggests that Equation 2 should be more generally expressed as

$$\frac{R_a}{R_b} = c \left(\frac{r_a}{r_b} \right)^x$$

where c is a parameter reflecting bias and x a fractional exponent reflecting the degree of undermatching.

The present work is not principally concerned with bias (which has been extensively discussed by de Villiers, 1977) but seeks to provide some basis for the prediction of undermatching. Although undermatching has been consistently found in most recent studies using *conc VI VI* schedules of food reinforcers (e.g., Davison & Hunter, 1976; Hunter & Davison, 1978; Lobb & Davison, 1975), as well as concurrent random interval schedules (Rodewald,

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1978), earlier discussions of undermatching had tended to attribute it to artifacts such as an inadequate changeover delay (Baum, 1974; de Villiers, 1977) or carryover effects from earlier schedule conditions (de Villiers). Baum, however, conjectured that true undermatching might result from a tendency on the part of the subject to "rescale the value of the rate of reinforcement according to a power function" (p. 232), although this suggestion was not elaborated into a formal theory.

The present work regards most undermatching as nonartifactual and shows that it is consistent with the well-documented tendency for aspects of the behavior of organisms to vary as a power function of scheduled time intervals. Furthermore, many of the VI schedules used in *conc* VI VI studies have a property which permits the degree of undermatching to be simply, and accurately, predicted.

Consider two concurrently available VI schedules, *a* and *b*, delivering rates of reinforcement, r_a and r_b . Assume that $r_a = mr_b$, where m is some constant, not necessarily an integer. This proportionality (m) is often arranged by programming the intervals that comprise each VI schedule as constant multiples of each other. Thus, if the time intervals comprising schedule *a* are $t_1, t_2 \dots t_n$, those comprising schedule *b* will be $mt_1, mt_2 \dots mt_n$. This arrangement is found in arithmetic VI schedules such as those used in the concurrent schedule studies of Davison and Hunter (1976), Hunter and Davison (1978), and Lobb and Davison (1975). It will also occur when VI schedules are arranged as recommended by Catania and Reynolds (1968, p. 381). In practice, the delivered rates of reinforcement may not exactly match the programmed minimum interreinforcement intervals, and therefore an exact correspondence between the two schedules such as that proposed above may not precisely hold. For simplicity, however, it will be assumed below that differences between programmed and delivered reinforcement rates are negligible.

The rates of reinforcement (r_a and r_b) on schedules *a* and *b* can be found by

$$r_a = \frac{1}{\frac{1}{n} \sum_{i=1}^n t_i} \quad (2a)$$

and

$$r_b = \frac{1}{\frac{m}{n} \sum_{i=1}^n t_i} \quad (2b)$$

where t_i is the i th of the interreinforcement intervals making up the schedule.

However, most of the available evidence suggests that the behavior of animals varies as a power function of any imposed temporal constraints, i.e.

$$f(t) = kt^x \quad (3)$$

where $f(t)$ is the value of some behavioral dependent variable which changes with the time constraint, t ; k is a constant; and x a fractional exponent. This conclusion is reached from the results of diverse experiments with both rats and pigeons which additionally suggest that the exponent in Equation 3 lies for most individuals between .7 and .9. Among the measures from which this value is derived are (a) latency to key peck after onset of a signal (Catania, 1970), (b) interresponse times on a spaced responding schedule (Catania), (c) the time to complete the whole (De Casper & Zeiler, 1974) or part (De Casper & Zeiler, 1977) of a fixed ratio, and (d) the postreinforcement pause on fixed-interval schedules of food for both rats and pigeons (Lowe, Harzem, & Spencer, 1979). Equation 3 can be regarded as reflecting some kind of organismic time estimation process (Catania), or simply as a transformation which must be applied to a temporal parameter of a schedule in order to yield a quantitative prediction of some aspect of behavior.

In view of the generality of Equation 3, it seems reasonable to assume that the interreinforcement intervals on a VI schedule are also transformed into "estimates," e.g.,

$$e(t_i) = kt_i^x \quad (4)$$

and that "estimates" of rates of reinforcement are derived from these, e.g.,

$$e(r_a) = \frac{1}{\frac{1}{n} \sum_{i=1}^n e(t_i)} \quad (5)$$

where $e(r_a)$ is the "estimate" of the reinforcement rate delivered by any schedule, *a*. If the time intervals comprising the concurrently pre-

sented schedules have a constant proportionality, m , then, using Equations 4 and 5

$$e(r_a) = \frac{1}{\frac{k}{n} \sum_{i=1}^n t_i^x}$$

and

$$e(r_b) = \frac{1}{\frac{km^x}{n} \sum_{i=1}^n t_i^x}$$

or, simplifying and rearranging,

$$\frac{e(r_a)}{e(r_b)} = m^x$$

If we assume that Equation 2 operates with $e(r_a)$ and $e(r_b)$ rather than rates as calculated by Equations 2a and 2b then

$$\frac{R_a}{R_b} = \frac{e(r_a)}{e(r_b)} = m^x$$

or, since $r_a = mr_b$

$$\frac{R_a}{R_b} = \left(\frac{r_a}{r_b} \right)^x \quad (6)$$

Since the exponent in Equation 4 is typically less than 1.0 Equation 6 represents undermatching. Taking logs of both sides of Equation 6, i.e.

$$\log \left(\frac{R_a}{R_b} \right) = x \log \left(\frac{r_a}{r_b} \right)$$

produces a straight line of slope x when log response ratios are plotted against log delivered reinforcement ratios. Thus, when VI schedules are arranged such that there is a constant proportionality between individual intervals at different VI values undermatching invariably results, and the degree of undermatching is exactly equal to the power function exponent. If this exponent is the same as that fitted in studies of temporal control the predicted degree of undermatching agrees well with obtained values, as noted by Davison and Tustin (1978).

The above derivation has proceeded by assuming that bias is absent. If there is a bias c such that $e(r_a)$ becomes $ce(r_a)$, substituting in Equation 5 and simplifying yields

$$\log \left(\frac{R_a}{R_b} \right) = x \log \left(\frac{r_a}{r_b} \right) + \log c$$

If *conc* VI VI schedules are arranged so that there is no constant proportionality between individual intervals the equations do not simplify as above, and the degree of undermatching is not exactly equal to the power function exponent. In general, however, the lower the value of the power exponent in Equation 4 the greater the degree of undermatching. For example, consider a VI 30-sec schedule composed of interreinforcement intervals 1, 7, 17, 30, 40, 50, and 65 sec presented concurrently with a VI 60-sec schedule with interreinforcement intervals 10, 14, 29, 60, 90, 100, and 117 sec. From Equation 2, the ratio responses to VI 30-sec/responses to VI 60-sec should equal 2.0. If the time intervals are transformed by Equation 4 and the resulting values substituted in Equation 5, the above ratio will be 1.87 if the exponent (x) in Equation 5 is .9, 1.65 if $x = .7$, and 1.45 if $x = .5$.

The above analysis produces "estimates" of the reinforcement rates by linear combination of transformed time intervals (e.g., Equation 5). In the case in which there is a constant proportionality between the time intervals composing each schedule, undermatching could be more simply derived by power transforming the ratio of the mean interreinforcement intervals, or the ratios of the i th intervals in each schedule. However, using the ratio of means is inconsistent with the view that individual time intervals are transformed, and using the ratios of the i th intervals involves arbitrary choice of i . Additionally, the procedure used in the above derivation (i.e., transformation of individual interreinforcement intervals by Equation 4 and substitution of resulting values in Equation 5) predicts undermatching even when there is no constant proportionality between the scheduled intervals.

The present account resembles that of Baum (1974) except that here interreinforcement intervals are power-transformed. Another suggestion by Baum, that undermatching results from poor discrimination between the schedules is also captured here since transforming interreinforcement intervals tends to reduce the discriminability of reinforcement rate differences if these are based on interreinforcement interval discriminations.

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